# Anomalous magnetic moment in parity-conserving $QED_3$

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## Abstract

In this article we derive the anomalous magnetic moment of fermions in (2+1)-dimensional parity-conserving QED$_3$, in the presence of an externally applied constant magnetic field. We use a spectral representation of the photon propagator to avoid infrared divergences. We also discuss the scaling with the magnetic field intensity in the case of strong external fields, where there is dynamical mass generation for fermions induced by the magnetic field itself (magnetic catalysis). The results of this paper may be of relevance to the physics of high-temperature superconductors.

# 1 Introduction

Three dimensional gauge theories find interesting physical applications to the physics of planar high-temperature superconductors and more general doped planar antiferromagnets [1]. On the other hand, three dimensional field theories may be used as toy models for four dimensional physics in many respects, for instance to get some information on the treatment of non perturbative effects of gauge theories [2].

Odd dimensional field theories are characterised in general by the anomalous breaking of the discrete symmetries of Parity (P) and/or Time reversal (T), and this may have profound physical consequences for the relevant systems. For instance there is a long and, in our opinion, still outstanding debate as to whether parity is broken spontaneously or anomalously by the ground state of high-temperature superconductors, whose physics may be modeled in some respect by three-dimensional gauge field theories.

In this last respect we mention that current theoretical models of high-temperature superconductors are based on the fact that such systems are characterised by nodal points in their surfaces, i.e. points in momentum space where some energy gap function vanishes. Linearisation about such points results in relativistic models of high temperature superconductivity, which have been proposed some time ago [3], and recently have been revived from a rather different perspective [4].

An important feature of the antiferromagnetic nature of the system, or the way the continuum field-theoretic limit is taken, is the doubling of the relevant fermion species which represent charged degrees of freedom in a spin-charge separation framework. Indeed, the continuum degrees of freedom of the resulting (2+1)-dimensional field theory are two 2-component fermions, $\Psi_1, \Psi_2$, coupled to a statistical gauge field, representing magnetic interactions in the underlying condensed-matter system, and interacting with external electromagnetic field, since the fermions (holons) represent charged excitations [1].

The presence of an even number of fermion species implies a parity-conserving model as far as the fermion mass is concerned. This is because in that case, parity can be defined in an extended way so as to exchange fermion species, thereby leading to a parity-invariant mass. The two compo-

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[1]In some other approaches [5] these fermions are electrically neutral, and it is the bosons in a spin-charge separation framework that are electrically charged. In the present article we shall not discuss such models.

nent fermions may be combined to a four-component spinor $\psi = (\Psi_1, \Psi_2)$, in which case a parity invariant mass term is simply $m\overline{\psi}\psi$. Depending on the details of the underlying statistical model one may also have more than one "flavour" of these four component spinors.

The dynamical generation of a parity -invariant mass for fermions seems to be preferred energetically due to the Vafa-Witten theorem [6], in the absence of externally applied fields. This defines then a P-invariant ground state. However, from a physical point of view high-temperature superconducting materials are strongly type II superconductors, which implies that when an external magnetic field is applied, the magnetic lines will penetrate the material in its superconducting phase significantly. As a consequence, it makes sense to consider the effects of an external magnetic field on holons in their superconducting state. This brings up the question as to whether a parity-violating mass can be generated dynamically under the influence of the external field, since in that case the Vafa-Witten theorem is inapplicable.

Experimentally there are indications that parity violation (in the form of resultant edge chiral currents in the superconducting planar materials) may occur in such circumstances. According to the models of [3, 7], however, a superconducting phase is characterised by massive charged fermions (holons) with a parity-conserving mass. In fact it has been argued that it is the nodal relativistic fermions that play a crucial role in the passage from the pseudogap to the superconducting phase [8]. Even in the presence of a magnetic field, which catalyzes the fermion mass generation, the parity conserving mature of the magnetically-induced fermion mass gap can be maintained in a consistent way [9].

It is therefore natural to inquire whether there is a possibility that parity is violated upon the application of external magnetic fields in the effective field theoretic continuum action of high temperature superconductors, as the experiments seem to indicate, but this violation is independent of the superconducting mass gap. It has been proposed in [10] that the answer to this question may come from the magnetically-induced anomalous magnetic moment of the fermions (holons), which from an effective action point of view contributes a non-minimal term that violates parity and time reversal, as a result of the fixed direction of the external magnetic field. In [11] a more elaborate situation of an induced magnetic moment for fermions has been calculated, in a case in which the gauge field is considered four dimensional but strongly anisotropic. These calculations showed that there is indeed an induced magnetic moment for the fermions, which violates parity and time

reversal, scaling with the externally applied field in a way that can be tested experimentally. However, those results were not complete in the sense that the definition for the magnetic moment used was the one that is usually applied to the weak field case.

In this article we complete the computation by considering the appropriate definition for the magnetic moment in both strong and weak fields. We consider only the case in which the field theory lives exclusively on a plane, and compute the corresponding anomalous magnetic moment for fermions in the case of magnetic fields applied perpendicularly to the plane where the fermions live. This computation has a field theoretic interest in its own right, independently of the possibility of applying the result to the physics of condensed-matter systems such as antiferromagnets and superconductors. As we shall see, the issue of the removal of infrared (IR) divergences by means of spectral decomposition of propagators is non trivial in this case. For these reasons we have written the article in a language suitable for field theorists, reserving a possible connection with condensed matter physics only for the conclusions. We intend to come back to a detailed condensed-matter analysis in a future publication.

The structure of the article is as follows: Section 2 presents the computation of the anomalous magnetic moment $\mu_B$ in the usual case, i.e. in the limit of vanishing external field. This computation follows the well-known derivation in 3+1 dimensions, but needs more care in order to deal with a logarithmic IR divergence. This is done by using a spectral representation of the dressed photon propagator. This method, originally set up in [12], cures the would-be IR divergence by taking into account a partial resummation of higher order quantum corrections and leads to a non-analytic dependence in the coupling constant. We discuss two separate cases: (i) weakly coupled gauge theories with a large (compared to the coupling) bare fermion mass, and (ii) strong coupling gauge models, with a small (compared to the coupling) dynamically generated mass of the fermions, in a large-$N_f$ treatment, with $N_f$ the number of flavours of four-component spinors. We also compare briefly our results with some relevant results on induced magnetic moments in parity violating gauge models of anyons. Section 3 deals with the computation of the anomalous magnetic moment in the presence of a strong external magnetic field. In this case we have to come back to the general definition of $\mu_B$ using the fermion self energy computed with the non-perturbative fermion propagator in the presence of an external magnetic field. We will show that the anomalous magnetic moment can be defined in the lowest Landau level,

provided we are in the rest frame of the massive fermion. $\mu_B$ also exhibits a logarithmic divergence that is cured with the same spectral representation method as in Section 2. We also consider in this case the scaling of the anomalous magnetic moment with the strong magnetic field. For this we consider, for the fermion mass, the dynamical mass $m_{dyn}$ that is generated by the external field, using previous works [9, 13] where the dependence of $m_{dyn}$ on the magnetic field was set up. Conclusions and a possible connection of our results with the physics of high temperature superconductors are presented in Section 4.

## 2    Magnetic moment in weak field

In the case of a weak external magnetic field $\vec{B}$, the magnetic moment $\mu_B$ is defined as the coefficient of the first order, linear, interaction between $\vec{B}$ and the spin of the fermion [14]. $\mu_B$ can be found by computing the quantum corrections to the vertex $\Gamma^\rho$ representing the interaction of $A_\mu^{ext}$ with the fermion $\psi$, which is taken as a 4-component fermion including the two flavours $\Psi_1$ and $\Psi_2$ (see fig. 1(a) [2]). In the limit where the momentum flowing from the external field vanishes, the on-shell vertex reads:

$$\Gamma^\rho_{on-shell}(p,q) = \mathcal{Z}\gamma^\rho - \frac{\mu_B}{2m}(p^\rho + q^\rho), \tag{1}$$

where $\mathcal{Z}$ is the quantum correction to the minimal coupling, $m$ is the fermion mass, and $g$ is the coupling to the dynamical gauge field [3]. In this expression,

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[2]It is understood that one should add to these (one-particle irreducible) graphs also one-particle reducible graphs (c.g. fig. 1(c)), where the external fermion lines get self energy corrections, expressing the effects of wave function renormalization to the external fermion polarisation tensors. Such corrections are proportional to $\gamma^\mu$ and do not contribute to the anomalous magnetic moment part of the vertex (1), which is linear in momenta $(p+q)^\mu$. This will always be understood in what follows, and it is valid for all ranges (strong, intermediate and weak) of *both* the gauge coupling and the external field. To be complete in our statements, therefore, we say that the sum of these graphs and those of fig. 1 yields formally the physically observable contributions to the anomalous magnetic moment.

[3]Throughout this work we keep $g$ different from the real (three-dimensional) electromagnetic coupling $e$. This allows for a straightforward extension of the present analysis to models of relevance to high temperature superconductors [3, 7, 4], where the statistical photon, distinct from the real electromagnetic one, expresses effectively magnetic interactions among the relevant degrees of freedom.

$p, q$ are the momenta of the incoming and outgoing fermion, i.e. we consider the limit $p \to q$. Using Dirac equation, the on-shell vertex (1) leads then to the expected anomalous magnetic moment term in the effective lagrangian

$$\frac{\mu_B}{2m} \overline{\psi} \sigma^{\rho\kappa} \psi \ \partial_\rho A_\kappa^{ext}, \tag{2}$$

which corresponds to a non-minimal parity-violating coupling generated by quantum effects.

Before proceeding with the relevant analysis we consider at this stage useful to point out that in our model the limiting relativistic velocity $v_F$ of the fermions and dynamical gauge bosons is not, from a physical viewpoint, the real speed of light $c$, but rather the velocity of the node of the fermi surface of an underlying microscopic condensed matter model [3, 7, 4] whose low energy effective theory near the node is described by the "relativistic" gauge model under consideration. For realistic systems [3, 7, 4, 5] one has (at most) $v_F \sim 10^{-4}c$, which means that in our system of units $v_F = 1$ we have $c \sim 10^4$ (at least). This implies that [7] in this model the real electron charge $\tilde{e}$ is related to the effective three-dimensional charge $e$ appearing in the model by

$$e = \tilde{e}/c \ , \tag{3}$$

which in turn means that the physically observable value of the anomalous magnetic moment should be given by rescaling the value $\mu_B$ in (2) by $1/c$:

$$\mu_B^{\text{phys}} = \frac{1}{c} \mu_B \tag{4}$$

This will always be understood throughout this work.

In what follows we shall distinguish two cases: (i) a bare fermion mass, and weak gauge coupling $g^2 \ll m$, (ii) a sufficiently strong gauge coupling $g^2$, responsible for the dynamical generation of fermion mass $m \ll g^2$. In case (ii) the mass scales with the weak magnetic field $B$, as shown in [15], as a result of the well-known magnetic catalysis phenomenon [16].

## 2.1   Weak Gauge Coupling and Bare Fermion Mass

We consider the Feynman gauge and assign the mass $M$ to the dynamical vector field to regularize the integrals involving an infrared divergence. The latter will then be avoided using a spectral representation of the photon propagator.

The one-loop correction to the vertex (c.f. fig. 1(a)) is, in the limit of zero incoming momentum from the external field,

$$\Gamma^\rho(p) = -ig^2 \int \frac{d^3k}{(2\pi)^3} \frac{\gamma^\mu(\not{p}+\not{k}+m)\gamma^\rho(\not{p}+\not{k}+m)\gamma_\mu}{[(p+k)^2-m^2]^2[k^2-M^2]}. \tag{5}$$

For on-shell fermions, we have $p^2 = m^2$, such that

$$\Gamma^\rho(p) = -\frac{2ig^2}{(2\pi)^3} \int_0^1 dx \int_0^{1-x} dy$$

$$\times \int d^3k \frac{\gamma^\mu(\not{p}+\not{k}+m)\gamma^\rho(\not{p}+\not{k}+m)\gamma_\mu}{\{[k+(x+y)p]^2-(x+y)^2m^2-(1-x-y)M^2\}^3}. \tag{6}$$

The vertex appears as $\overline{\psi}\,\not{V}\psi$ in the Lagrangian, such that the momentum $\not{p}$, when appearing on the left or the right of the group of matrices $\gamma^\mu, \gamma^\nu, \gamma^\rho$ can be replaced by $m$ since for on-shell fermions we have $\not{p}\psi = m\psi$. One obtains then for the terms proportional to $p^\rho$

$$\frac{4\pi i \times 2ig^2}{(2\pi)^3} \int_0^1 dx \int_x^1 dz \int_0^\infty dq_E \; q_E^2 \frac{8z(1-z)mp^\rho}{[q_E^2+z^2m^2+(1-z)M^2]^3}, \tag{7}$$

where $q_E = k_E + zp_E$ is the Euclidean momentum and $z = x + y$. The integration over $q_E$ gives finally the following expression for the magnetic moment

$$\mu_B^{(0)}(m, M) = \frac{g^2m^2}{2\pi} \int_0^1 dx \int_x^1 dz \frac{z(1-z)}{[z^2m^2+(1-z)M^2]^{3/2}}. \tag{8}$$

Here the subscript $(0)$ stands for the naive magnetic moment, where the infrared divergence at $M = 0$ has not yet been taken into account. We will come back to this divergence after integrating over $z$ and $x$ for finite $M$.

First, we can see that in the case $m = 0$ and $M \neq 0$, the integration over $x, z$ is convergent and the anomalous magnetic moment vanishes:

$$\mu_B^{(0)}(0, M) = 0. \tag{9}$$

But we are more interested by the case $m \neq 0$ and $M = 0$. For this we first keep $M \neq 0$. We obtain then: $\mu_B^{(0)} = (g^2/2\pi m)(J_1 + J_2)$ where, in the limit $\beta \to 0$, $J_1$ has a finite value and $J_2$ contains an infrared logarithmic

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divergence. We have, for any non-vanishing value of the parameter $\beta = M^2/m^2$,

$$J_1 = \int_0^1 dx \left\{ \frac{3-\beta}{2-\beta/2} \left( 1 - \frac{\beta}{2} - \frac{x-\beta/2}{\sqrt{x^2+(1-x)\beta}} \right) + \right.$$
$$\left. \beta - 1 - \ln \left( \frac{4-\beta}{2x-\beta+2\sqrt{x^2+(1-x)\beta}} \right) \right\} =$$
$$\frac{3\beta+3\sqrt{\beta}-4}{\sqrt{\beta}+2} - \frac{\beta}{2} \ln \left( 1 + \frac{2}{\sqrt{\beta}} \right)$$
$$J_2 = \int_0^1 dx \frac{1-\beta}{\sqrt{x^2+(1-x)\beta}} = (1-\beta) \ln \left( 1 + \frac{2}{\sqrt{\beta}} \right). \tag{10}$$

To avoid the divergence of $J_2$ in the limit $M \to 0$ we proceed as in [12], where the authors compute higher-loop corrections to the photon and fermion propagators in massless $QED_3$. Instead of the bare photon propagator that was used in Eq.(5), we consider the following spectral representation of the dressed propagator for massless gauge boson

$$D_{\mu\nu}(k) = g_{\mu\nu} \int_0^\infty dM \frac{\rho(M)}{k^2-M^2}, \tag{11}$$

where the weight $\rho(M)$ includes the quantum corrections. It is known that the one-loop vacuum polarization is [17]

$$\Pi_{\mu\nu}(k) = \frac{g^2}{4\pi} \left( g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \left\{ 2m + \frac{k^2-4m^2}{k} \sin^{-1} \left( \frac{k}{\sqrt{4m^2+k^2}} \right) \right\}, \tag{12}$$

where $k = \sqrt{k^2}$, such that, in the limit of small momentum $k$

$$D_{\mu\nu}(k) = \frac{g_{\mu\nu}}{k^2 - \frac{g^2}{6\pi}k} \tag{13}$$

Note that there is no branch cut problem in the complex plane $k$: the propagator (13) is an approximate one and the only pole that occurs when taking the full polarization tensor (12) into account is for $k = 0$. Note also the non-perturbative nature of the infrared quantum corrections since $g^2 k \gg k^2$ when $k \to 0$.

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To take into account the correction (13) for small momenta, which is sufficient to cure the IR divergence, the function $\rho(M)$ should be taken as follows:

$$\rho(M) = \frac{\frac{g^2}{3\pi^2}}{M^2 + \left(\frac{g^2}{6\pi}\right)^2}. \tag{14}$$

Taking higher order correction with this method cures the infrared divergence of the magnetic moment since the former is logarithmic and thus integrable. The expression for the anomalous magnetic moment for massless gauge boson is thus given by

$$\begin{aligned} \mu_B(m,0) &= \int_0^\infty dM \rho(M) \mu_B^{(0)}(m,M) \tag{15} \\ &= \frac{g^2}{2\pi m} \int_0^\infty dM \rho(M) \left\{ J_1(m,M) + J_2(m,M) \right\}. \end{aligned}$$

Since the weight $\rho$ is normalized, i.e. $\int_0^\infty dM \rho(M) = 1$, we have

$$\begin{aligned} \mu_B(m,0) &= \frac{g^2}{2\pi m} \left\{ J_1(m,0) + \int_0^\infty dM \rho(M) J_2(m,M) \right\} + \mathcal{O}(g^2/m)^2, \\ &= -\frac{g^2}{\pi m} + \frac{g^2}{2\pi m} \int_0^\infty dM \rho(M) J_2(m,M) + \mathcal{O}(g^2/m)^2, \tag{16} \end{aligned}$$

where $\mathcal{O}(g^2/m)^2$ are higher order terms in $g^2/m$. An expansion in powers of $g^2/m$ leads then to

$$\int_0^\infty dM \rho(M) J_2(m,M) = -\ln\left(\frac{g^2}{12\pi m}\right) + \mathcal{O}(g^2/m), \tag{17}$$

where we used the identity

$$\int_0^\infty dy \frac{\ln y}{1+y^2} = 0. \tag{18}$$

Finally, the divergence-free anomalous magnetic moment is, in the case of massless dynamical gauge field and massive matter field $m \neq 0$

$$\mu_B(m,0) = -\frac{g^2}{\pi m} + \frac{g^2}{2\pi m} \ln\left(\frac{12\pi m}{g^2}\right) + \mathcal{O}(g^2/m)^2. \tag{19}$$

Note that, as expected from this spectral representation method, the would-be infrared divergence is cured by the non-analytical dependence $\varepsilon \ln \varepsilon$ in the dimensionless coupling $\varepsilon = g^2/m$. It is understood that the rescaling (4) is in operation if we wish to compute the physically measurable value.

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## 2.2 Strong Gauge Coupling and Dynamically Generated Fermion Mass

In the remainder of this section we would like to discuss briefly the case where $g^2 \gg m$, which is the case where one has a dynamically generated mass. This case is also of great physical interest due to its direct application to high temperature superconducting models [3, 7]. A complete treatment of this case is not yet available, nevertheless one can get a consistent and satisfactory for our purposes treatment within the so-called large $N$-approximation [17, 18, 19].

To this end, we assume there are $N_f$ four-component fermionic flavours, and one fixes the dimensionful gauge coupling of the statistical gauge field (not the real electromagnetic one) as: $\alpha = N_f g^2/8$. The large-$N$ treatment follows by letting $N_f \rightarrow \infty$ while keeping $\alpha$ fixed (large but finite) [4]. Notice that in this way one introduces the ratio of the bare coupling $g^2$ over the fixed scale $\alpha$, $g^2/\alpha \propto 1/N_f \ll 1$ as a small dimensionless expansion parameter in the analysis, and hence one can follow the spectral representation method described above which is strictly valid for weak (dimensionful) coupling $g^2$. However, we stress that, as compared with the dynamically generated fermion mass $m$, the fixed scale $\alpha/m \gg 1$, hence this is not an expansion parameter.

To leading order in resummed one loop (leading $1/N_f$) analysis one obtains a modified statistical photon propagator:

$$\Delta_{\mu\nu}(k^2) = \frac{g_{\mu\nu}}{k^2 \left(1 - \Pi(k^2)\right)} \tag{20}$$

with $\Pi(k^2) \sim c_1 \alpha/k + \mathcal{O}(1/N_f)$, to leading order in $1/N_f$ where $c_1$ is a numerical constant ($N_f$ dependent) computed in [18]. This implies a dressed photon propagator with softened infrared behaviour which is formally similar to (13) upon the substitution of $g^2$ by the fixed scale $\alpha$, however here the result resums one loop corrections, and hence is an improved version appropriate for strong coupling.

The dressed fermion propagator is:

$$S(p) = \frac{1}{A(p)} \frac{i \not{p} + m(p)}{p^2 - m^2(p)} \tag{21}$$

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[4]In the physical models of [3, 7] $N_f = 2$, which is within the range of $N, N_c \simeq 3 - 4$ [18] that allows dynamical fermion mass generation $m \ll \alpha$.

The fermion wavefunction renormalisation $A(p)$ has been worked out in large-$N$ treatments [18], and in fact there are ambiguities as to whether it should vanish as $p \to 0$ [1, 18]. However, as we shall discuss below, the explicit form of $A(p)$ will not be relevant for our purposes, due to the fact that the $A(p)$ factors cancel out in the expression for the statistical-photon dressed electromagnetic vertex function, as a result of the use of dressed vertex functions for the statistical photon [18], $\Gamma_s^\mu(p, q, k) = \frac{A(p^2) + A(q^2)}{2} \gamma^\mu$, where p,q refer to fermion lines.

Indeed, the electromagnetic vertex function used for the computation of the magnetic moment in the model with a statistical and a real photon, reads in the infrared regime, where $p \simeq q$, $k \to 0$, which is the region responsible for the dominant contributions to the magnetic moment:

$$\Gamma_{\text{em}}^\rho = \gamma^\rho - i \frac{c_2}{N_f} \alpha \int \frac{d^3 k}{(2\pi)^3} A(p) \gamma^\mu S(p-k) \gamma^\rho S(p-k) \gamma_\mu A(p) \frac{1}{k^2(1 - \Pi(k^2))} \tag{22}$$

where $c_2$ a numerical coefficient, of $\mathcal{O}(1)$, and above we have explicitly demonstrated the $1/N_f$ dependence of the corrections to the electromagnetic vertex, which is due to the fact that, in contrast to the case of fermion loops in the $1/N_f$-corrections to the statistical photon propagator, there is no summation over internal fermion lines at the vertex involving the real electromagnetic field. Thus the corrections are proportional to the coupling $g^2 = 8\alpha/N_f$ and *not* to the fixed scale $\alpha$. In the infrared limit $p - k \to p$ one can easily infer from (21) the above-mentioned cancellation of wavefunction renormalisation $A(p)$ factors. Notice that this is due to the fact that the real electromagnetic vertex in (22) is not dressed, in contrast with the statistical photon vertex, since $e^2 \ll \alpha$.

In the approximation where $m(p) \simeq m(0) \equiv m$ one then obtains an expression for the vertex formally similar to (5), with the improved photon propagator (20), which is obtained from (13) upon replacing $g^2/6\pi$ by $c_1\alpha$. Then, applying the spectral representation technique, used previously (11),(14), with the above replacement, we arrive at the expression (15) for the magnetic moment, with the important difference that now it is proportional to $\alpha/mN_f \ll 1$, as $N_f \gg 1$, where $m$ denotes the dynamically generated mass [18]. We note here that the analysis of [15], using also preliminary quenched lattice studies, have indicated a quadratic scaling of the dynamical fermion mass with the weak field $B$, for strong and intermediate statistical gauge couplings $g$ in the model, $m/g^2 = \text{const} + \mathcal{O}(\frac{e^2 B^2}{g^8})$, for $eB \ll g^4$,

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but this still need to be confirmed by more complete lattice analysis using fully dynamical fermions. However, the analytic results (15) on a non trivial $\mu_B$ in the limit of vanishing field intensity $B \to 0$ derived above for strong and intermediate statistical gauge couplings are expected to hold, at least qualitatively.

As remarked earlier, for weakly coupled gauge theories, the spectral representation analysis of [12], developed through one-loop Feynman graphs, proves sufficient in dealing with the infrared infinities of higher-order (in $g^2$) graphs, beyond one loop. This is due to the smallness of the coupling $g^2$, as compared with a typical momentum (or mass) scale in the problem, which allows for a consistent perturbation expansion over $g^2/m$ to be developed, that softens the infrared infinities of the higher-order graphs to logarithmic.

Fortunately, such a cure of higher-order infrared infinities is also true for our large-$N$ strongly coupled gauge theory studied in this subsection. Indeed, in this case, although $\alpha/m \gg 1$, where $m$ is the dynamically generated fermion mass [17, 18], and hence is not a good expansion parameter, infrared divergent higher-order graphs are of order $1/N_f^2$, and hence subleading in the limit $N_f \gg 1$. The self-consistency of the leading order large-$N$ treatment [18], then, can be used as an argument supporting the qualitative conclusions drawn from the leading order analysis above, as far as the magnetic moment calculation is concerned [5]. Of course, a more complete method in dealing with the problem is still needed for quantitatively detailed computations to be in place [6], but this falls beyond the scope of the present work.

The anomalous magnetic moment $\mu_B$ (15) is well-defined from the point of view of (infrared or ultraviolet) divergences, and can be computed numerically; In a large-$N$ treatment, to leading order in $1/N_f$ [18], and for weak external magnetic fields, the dynamical fermion mass is considered as constant, although, as we mentioned above, in actual situations there are

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[5]In fact, studies of higher-order $1/N^2$ corrections [19] have shown that their effects, at least on the dynamical mass generation, are small (of the order of 20%), so that reliable qualitative information on the phase structure of the model can be inferred from a leading order $1/N$ analysis.

[6]This is especially because of unresolved ambiguities as far as the behaviour of the fermion wavefunction renormalization $A(p)$ in the infrared limit is concerned. For instance, improved Schwinger-Dyson approaches [1] combining large-$N$ treatments with the use of pinch techniques [20] show a singular behaviour of $A(p)$ as $p \to 0$, in contrast with conventional large-$N$ treatments [18], where $A(p)$ is either regular or goes to zero in this limit.

corrections to this constant value, exhibiting a non-trivial (quadratic) scaling with the magnetic field [15]. From (15) we then obtain for the observable magnetic moment:

$$\mu_B(N_f) = \frac{4\alpha^2}{cN_f m^2 \pi^2} \int_0^\infty dy \frac{1}{y^2 + (\frac{\alpha}{2m})^2} \left( J_1(y^2) + J_2(y^2) \right) \qquad (23)$$

where $y = M/m$, and $J_{1,2}$ have been defined in (10). In (23) $c$ is the real speed of light, which in units of the fermi velocity we are working on in this paper is $c \sim 10^4$. As stressed in the beginning of section 2 (c.f. (4)), this suppression factor is essential to yield the correct order of magnitude of the physical magnetic moment in our system of units, which is defined as the response of the system to an external electromagnetic field, whose magnitude is normalised with respect to the real speed of light.

The integration over $y$ is finite, as can be easily seen, in *both* the ultraviolet and infrared regimes of $y$ [7], and the result can be computed numerically. Formally, therefore, $\mu_B$ is of order $1/N_f$, which is small as long as $N_f \gg 1$.

The problem is that in practice $\alpha/m \gg 1$, since $N_f \sim 2$, and in large N treatments $m \sim \alpha e^{2+\delta-2\pi/\sqrt{N_c/N_f-1}}$, where $\delta$ is an arbitrary phase, not determined to leading order in $1/N$ expansion [17, 18], $3 < N_c < 4$, and $N_f < N_c$ for dynamical mass generation. In order to obtain a physically consistent result within the $1/N_f$ expansion, the anomalous magnetic moment must be a small number, much smaller than 2S (=1 for fermions with spin S=1/2, where 2 is the gyromagnetic ratio), which, however, does not follow in practice from the leading order results in $1/N_f$, as far as the order of $m$ is concerned [17, 18, 19], unless one selects unnaturally large values of the phase

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[7]The integrals are manifestly infrared ($y \to 0$) finite, the potentially dangerous ultraviolet ($y \to \infty$) divergent terms coming from the $\mathcal{O}\left(\beta/(\sqrt{\beta}+2)\right)$ and logarithmic terms of $J_1$ in (10) upon integration over $\sqrt{\beta} \equiv y$, weighted by terms of the form $1/(y^2 + (\alpha/2m)^2)$. The dangerous terms are of the form: $I_1 + I_2$, where $I_1 = 3\int_0^\infty dy \frac{y^2}{(y^2+(\alpha/2m)^2)(y+2)}$ and $I_2 = \int_0^\infty dy \frac{y^2}{2(y^2+(\alpha/2m)^2)} \ln\left(\frac{y}{y+2}\right)$. To treat their ultraviolet divergent parts properly, we make use of the identity (18) in $I_2$, which yields for the divergent parts of $I_2$: $I_2^{(div)} = \frac{1}{2}\int_{y\to\infty} dy \ln\left(1 - \frac{2}{2+y}\right) \simeq -\int_{y\to\Lambda} dy \frac{1}{y} \sim -\ln\Lambda$, with $\Lambda$ an ultraviolet cutoff for (the pure number variable) $y$. In a similar spirit, the ultraviolet divergent parts of $I_1$ is treated properly by first changing integration variable $y \to x = y^3$, and then approximating the denominators by going to large $x$. This yields $I_1(div) = \int_{x\to\Lambda} dx/x = \ln\Lambda$, using the same ultraviolet cutoff $\Lambda$ for pure numbers. Therefore, the ultraviolet infinities cancel out exactly in $I_1 + I_2$, yielding an overall finite result for $\mu_B(N_f)$, in a large-N framework.

$\delta$. This is the main reason why one should go beyond the large-N treatment in order to get quantitatively reliable results for $\mu_B$, e.g. via pinch [20, 1] or other non-perturbative techniques, which however lies beyond our scope here. For us, the above large-$N$ computation of $\mu_B$ demonstrates (formally) a finite subleading result, of order $1/N_f$ in the limit of $N_f \gg 1$ and vanishing magnetic field, which is sufficient for our purposes in this work.

For our purposes, therefore, we can borrow at this stage results from lattice treatments [15] of QED$_3$, for the case of $N_f = 2$, according to which $m/\alpha \sim 0.48$, which also agrees in order of magnitude with the upper bound for the dynamically generated fermion mass determined by the pinch technique approach [1]. Substituting this value into (23) we then obtain an estimate of the induced anomalous magnetic moment: $\mu_B^{\mathrm{phys}}(N_f = 2) = \mathcal{O}(10^{-2}) < 1$ for the physical value $N_f = 2$. Notice that the smallness of the physically observable anomalous magnetic moment justifies the large-$N$ approximation in our model, and is due to the large value of the real speed of light $c \sim 10^4$, which furnishes additional suppression factors in the realistic case where the fermion flavours are $N_f = 2$. In fig. 2 we plot the induced anomalous magnetic moment (23) versus $\alpha/m$, for vanishing external field. The result points towards a quadratic dependence on this parameter $\mu_B \propto (\alpha/m)^2$, at least in the regime of interest in this work.

## 2.3   Comparison with an Anyon Model

Before closing this section we would like to contrast this result on the anomalous magnetic moment within a large-$N$ treatment of our parity invariant QED$_3$-like model, with a corresponding computation in an anyonic model in three-dimensions [21]. There, the authors using again a spectral representation of the photon propagator, obtain the anomalous magnetic moment to leading order in $1/N$ for their model, which is different from ours in that it consists only of a single Abelian gauge field interacting with $2N+1$ fermion species, in the presence of an Abelian Chern-Simons (CS) term for the gauge field, with a coefficient $\kappa$. In such a case, with intrinsic parity violation due to the dynamical CS term, the anomalous magnetic moment is essentially determined by the parity-violating parts of the dynamical gauge boson propagator, which are absent in our parity conserving case. The important thing the authors of [21] find is that, in the limit where the coefficient $\kappa$ of the CS term is very large, the induced corrections to the magnetic moment are of order $1/m\kappa$, where $m$ is the bare fermion mass in their model. In fact,

the result of the large-$N$ computation of [21] indicates a quantum corrected magnetic moment $\mu = \frac{1}{m}(\frac{1}{2} + \frac{1}{\kappa})$, which, with $S = \frac{1}{2} + \frac{1}{\kappa}$ identified as the spin of the anyon field [22], implies an exact (to leading order in $1/N$) gyromagnetic ratio $g = 2$ for anyons. This result is in agreement with general arguments [22] that the gyromagnetic ratio of an anyon system should remain 2 to all orders in a quantum treatment of localised anyon fields. Indeed, in the limit where $e^2 \kappa \gg m$, where $e$ is the gauge coupling, the anyon is viewed as a point charge, surrounded by a gauge field cloud of size $1/e^2 \kappa$, which collapses onto the charge, and such a system of charge-cloud composites display a fractional statistics.

In our case, things are entirely different. We have no anyonic fields to start with, hence no parity violating parts in the dynamical gauge boson propagator, and our fermions are ordinary fermions, exhibiting non trivial, but $1/N_f$ suppressed, corrections to their gyromagnetic ratio (23), responsible for the appearance of a parity violating term in the effective lagrangian (2). This term, however, vanishes when the external field is turned off. This is therefore not an intrinsic effect, like the one induced by the parity violating CS term in the anyon case, but an extrinsic effect, which is induced by an external electromagnetic field that has no dynamics, and certainly no CS term. Of course the direction of the magnetic field breaks time reversal, and hence parity in our relativistic system (due to CPT theorem), but this is an extrinsic breaking. The induced anomalous magnetic moment, although non zero when the field is turned off, however, does not couple to the statistical gauge field so as to imply non trivial interactions of a permanent parity violating nature (after the switching off of the field). The effects go away when the external magnetic field is switched off. Another important point to notice is that, unlike the anyon case, the sign of the induced magnetic moment is the same for all $2N_f$ two-component fermionic flavours, while their masses alternate sign ($N_f$ two-component fermions have masses $m$ and the remaining $N_f$ have masses $-m$). This completes our discussion on the comparison of our results with the anyonic case.

Our analysis in this work deals with parity invariant QED$_3$ models with one type of statistical gauge coupling among the fermions. This has applications to one approach to high temperature superconductivity [4]. Similar conclusions, however, are expected to hold in other parity invariant effective gauge theories of high temperature superconductors of the so-called $\tau_3$-QED type [3, 7], where the abelian statistical gauge field is the U(1) unbroken subgroup of a spin SU(2) local gauge group, and couples to the pertinent

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fermion species with opposite sign couplings, expressing the underlying antiferromagnetic structure of the microscopic model. We reserve a detailed discussion of such issues for a forthcoming publication.

This completes our discussion on the weak magnetic field case. We next proceed to discuss the effects of a strong external field on induced corrections to the magnetic moment in our model, which as we shall show scale non-trivially with the external field.

## 3   Magnetic moment in strong field

If we consider a strong external magnetic field, the magnetic moment is defined by computing the fermion self energy [23] (see fig. 1(b) [8]). For a magnetic field $B$ (and coupling $e$) in the direction perpendicular to the plane, the fermion self energy is then connected to the anomalous magnetic moment $\mu_B$ by the relation (apart from some phase factors, as will be seen further on)

$$\Sigma_{on-shell} = \delta m + \mu_B \frac{|eB|}{2m} \gamma^0 \left(1 + \cdots\right) \tag{24}$$

where $\delta m$ corresponds to the correction to the mass and the term in parentheses denotes the projection of the spin on the axis of the external magnetic field [16], with the dots representing terms including $\gamma^1, \gamma^2$. Note that both $\mu_B$ and $\delta m$ depend on $|eB|$. In this definition one can recognize the anomalous magnetic moment term leading to the Pauli equation in the non-relativistic limit (we remind the reader that, in our approach, its physical value is determined by the rescalings (3), (4)). Then we can compute $\Sigma_{on-shell}$ as a loop expansion, using for the fermion propagator the proper-time representation set up by Schwinger [24]. This representation allows to take into account the non-perturbative interaction of the fermions with the external magnetic field. In the weak field limit, this interaction is perturbative and consists only in one photon propagator insertion, i.e. the definition of $\mu_B$ using the vertex (1) is recovered.

In the case of a strong magnetic field $|eB| >> m^2$, where $m$ is the fermion mass, it is sufficient to consider fermions in the lowest Landau level (LLL)

---

[8]We remind the reader that our comments in the beginning of section 2, on the addition to these irreducible loop corrections of reducible graphs expressing wavefunction renormalization of the external fermion lines (c.g. fig. 1(c)), but not contributing to the magnetic moment, remain intact in the strong-field case as well.

only. We will discuss later the question of definition of the anomalous magnetic moment in the LLL.

We consider the gauge $A_\mu^{ext}(x) = (0, -Bx_2/2, Bx_1/2)$. The LLL fermion propagator is [16]:

$$
\begin{aligned}
S^L(x,y) &= \exp\{iex^\mu A_\mu^{ext}(y)\}\tilde{S}(x-y) \\
\text{with } \tilde{S}^L(p) &= i\exp\left\{-\frac{p_\perp^2}{|eB|}\right\}\frac{p_0\gamma^0 + m}{p_0^2 - m^2}(1 - i\gamma^1\gamma^2\text{sign}(eB)), \quad (25)
\end{aligned}
$$

where $p_\perp = (p_1, p_2)$. From now on, unless explicitly stated, we take sign$(eB)$=1.

The one-loop fermion self energy is given by [25]

$$
\begin{aligned}
\Sigma(x,y) &= ig^2\gamma^\mu S(x,y)\gamma^\nu D_{\mu\nu}(x,y) \\
&= e^{iex^\mu A_\mu^{ext}(y)}\int \frac{d^3p}{(2\pi)^3}e^{ip(x-y)}\tilde{\Sigma}(p), \quad (26)
\end{aligned}
$$

where $D_{\mu\nu}$ is the bare photon propagator and

$$
\tilde{\Sigma}(p) = ig^2\int \frac{d^3k}{(2\pi)^3}\gamma^\mu\tilde{S}(k+p)\gamma^\nu D_{\mu\nu}(k). \quad (27)
$$

Note that, due to the projector $(1 - i\gamma^1\gamma^2)$, the propagator $\tilde{S}^L(p)$ given in Eq.(25) has no inverse. For this reason, in the studies of dynamical symmetry breaking by a magnetic field in the LLL approximation [16], one cannot use a self-consistent Dyson-Schwinger equation involving the inverse of fermion propagator. In our case, since we are not dealing with a self-consistent equation but we are computing a one-loop correction, we can use the expression (26) which involves $S$ only. From this last equation we have

$$
\begin{aligned}
\Sigma(p,q) &= \int d^3x d^3y e^{-ipx-iqy}\Sigma(x,y) \\
&= \int d^3y e^{-iy(p+q)}\tilde{\Sigma}(p - eA^{ext}(y)), \quad (28)
\end{aligned}
$$

which is the correct expression of the fermion self energy in momentum representation. For the anomalous magnetic moment though, it is enough to consider the computation of $\tilde{\Sigma}_{on-shell}$, since $\mu_B$ is defined as a proportionality constant which is not influenced by the phase gauge-dependent factor $\exp\{iexA^{ext}(y)\}$, as can be seen from Eq.(26).

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Taking into account Eqs.(25), (27), we obtain in the Feynman gauge

$$\tilde{\Sigma}(p) = -ig^2 \int \frac{d^3k}{(2\pi)^3} \exp\left\{-\frac{(k_\perp + p_\perp)^2}{|eB|}\right\}$$
$$\times \frac{\gamma^\mu[(k_0 + p_0)\gamma^0 + m](1 - i\gamma^1\gamma^2)\gamma_\mu}{[(k_0 + p_0)^2 - m^2][k_0^2 - k_\perp^2 - M^2]}, \quad (29)$$

where, as in the previous section, we assign the mass $M$ to the photon.

This last expression can be written, when $\tilde{\Sigma}$ is on-shell,

$$\tilde{\Sigma}_{on-shell} = \frac{g^2 m}{(2\pi)^3} \int d^2 k_\perp \exp\left\{-\frac{k_\perp^2}{|eB|}\right\} \quad (30)$$
$$\times \int_0^1 dx \int_{-\infty}^{+\infty} dk_3 \frac{\gamma^\mu[(1-x)\gamma^0 + 1](1 - i\gamma^1\gamma^2)\gamma_\mu}{\{k_3^2 + x^2 m^2 + (1-x)(k_\perp^2 + M^2)\}^2},$$

where $ik_3 = k_0 + xp_0$ and the on-shell condition ($p_0 = m, p_\perp = 0$) was used (we remind that the energy of the $n$th Landau level satisfies $p_0^2 = m^2 + 2n|eB|$). This on-shell condition is not Lorentz invariant and this is the price to pay to define the anomalous magnetic moment in the LLL. It has indeed been pointed out in [23, 26, 27] that no anomalous magnetic moment in the LLL can be defined in a Lorentz invariant way. These authors used the equation of motion of the fermion for the on-shell condition, which is obviously Lorentz invariant. As a consequence, all reference to $\gamma^0$ disappears in the LLL and they can only define the mass shift. In our case, this mass shift corresponds to the term which remains after taking the trace of Eq.(30).

The integration over $k_3$ is straightforward and gives

$$\tilde{\Sigma}_{on-shell} = \frac{g^2 m}{16\pi^2} \int d^2 k_\perp \exp\left\{-\frac{k_\perp^2}{|eB|}\right\} \quad (31)$$
$$\times \int_0^1 dx \frac{\gamma^\mu[(1-x)\gamma^0 + 1](1 - i\gamma^1\gamma^2)\gamma_\mu}{\{x^2 m^2 + (1-x)(k_\perp^2 + M^2)\}^{3/2}}.$$

One can see that, in the limit $M \to 0$, a logarithmic IR divergence will occur. This divergence will be treated in the same way as in the previous section. To this end consider first the naive magnetic moment derived from the definition (24). The quantity $\tilde{\Sigma}_{on-shell}$ can be written as

$$\tilde{\Sigma}_{on-shell} = \delta m + \mu_B \frac{|eB|}{2m}\gamma^0(1 + 3i\gamma^1\gamma^2)$$
$$= \delta m - \mu_B \frac{|eB|}{2m}\gamma^0 \left([1 - i\gamma^1\gamma^2\text{sign}(eB)] - 2[1 + i\gamma^1\gamma^2\text{sign}(eB)]\right) (32)$$

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Note that in the last line we have re-written the right hand side in such a way so as to make clear that, when one calculates *on-shell* quantities entering the expression for the anomalous magnetic moment (see below), terms proportional to $1 + i\gamma_1\gamma_2\text{sign}(eB)$ do not contribute, because in the LLL approximation the spin is polarised along the magnetic field [16].

The anomalous magnetic moment $\mu_B$ is then

$$
\begin{aligned}
\mu_B^{(0)}(m, M) &= \frac{g^2 m^2}{8\pi} \int_0^\infty du\ e^{-u} \\
&\times \int_0^1 dx \frac{1 - x}{\{x^2 m^2 + (1 - x)(u|eB| + M^2)\}^{3/2}}.
\end{aligned}
\tag{33}
$$

We first note that in the case $m = 0$ and $M \neq 0$, the integration over $x$ converges and the final result for the magnetic moment gives $\mu_B^{(0)}(0, M) = 0$.

In the other situation, where $m \neq 0$ and $M \to 0$, we first consider $M \neq 0$ and write

$$
\mu_B^{(0)}(m, M) = \frac{g^2}{8\pi m} \int_0^\infty du e^{-u} \int_0^1 dx \frac{1 - x}{\{x^2 + (1 - x)(\alpha u + \beta)\}^{3/2}},
\tag{34}
$$

where $\alpha = |eB|/m^2 >> 1$ and $\beta = M^2/m^2$.

We first perform the integration over $u$, using the fact that $\alpha >> 1$: we can approximate

$$
\begin{aligned}
&\int_0^\infty du e^{-u} \frac{1 - x}{\{x^2 + (1 - x)(\alpha u + \beta)\}^{3/2}} \\
&= \int_0^1 du \frac{1 - x}{\{x^2 + (1 - x)(\alpha u + \beta)\}^{3/2}} + \mathcal{O}(1/\alpha)^{3/2} \\
&= \frac{2/\alpha}{\sqrt{x^2 + (1 - x)\beta}} + \mathcal{O}(1/\alpha)^{3/2}.
\end{aligned}
\tag{35}
$$

The remaining integral over the Feynman parameter $x$ gives then for any non-vanishing $\beta$

$$
\mu_B^{(0)}(m, M) = \frac{g^2 m}{4\pi|eB|} \ln\left(\frac{2m + M}{M}\right) + \mathcal{O}(m/\sqrt{eB})^3.
\tag{36}
$$

Finally, the divergence-free anomalous magnetic moment $\mu_B(m, 0)$ is obtained with the same method as in the previous section:

$$
\mu_B(m, 0) = \int_0^\infty dM \rho(M) \mu_B^{(0)}(m, M),
\tag{37}
$$

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where $\rho(M)$ is given by Eq.(14). We neglect here the influence of the magnetic field on the fermion loop, which corresponds to higher order contributions.

For the integration over $M$, we consider two cases which are relevant:

- in the limit $g^2 << m$:

$$\mu_B^{(1)}(m,0) = \frac{g^2 m}{4\pi |eB|} \ln\left(\frac{12\pi m}{g^2}\right) + \mathcal{O}(g^2/m); \tag{38}$$

- in the limit $m << g^2$:

$$\mu_B^{(2)}(m,0) = \frac{6m^2}{\pi |eB|}\left\{1 - \ln\left(\frac{12\pi m}{g^2}\right)\right\} + \mathcal{O}(m/g^2). \tag{39}$$

Note that among the three mass scales $(\sqrt{|eB|}, g^2, m)$, $\sqrt{|eB|}$ is the largest one, such that $\mu_B$ is always perturbative. Once again, we remind the reader that in order to give the physically correct order of magnitude, at least for potential applications to the theory of high temperature superconductivity, we should use the rescaling (3),(4).

We shall conclude this section by discussing the scaling of the anomalous magnetic moment $\mu_B$ with the externally applied magnetic field $B$. It was found in [9] that the fermion mass generated dynamically in the presence of a strong magnetic field is

$$m_{dyn} \simeq Cg^2 \ln\left(\frac{\sqrt{|eB|}}{g^2}\right), \tag{40}$$

where $C$ is a constant. This dependence has been seen numerically [13], where it was found that $C \simeq 0.06$.

Let us define the dimensionless magnetic strength $b = \sqrt{|eB|}/g^2$. The lowest Landau level approximation assumes that $m_{dyn} << \sqrt{|eB|}$, which from Eq.(40) and in terms of $b$ reads

$$C \ln b << b. \tag{41}$$

This is always the case, independently of the strength of the coupling $g$. Making the identification $m = m_{dyn}$ in the integral (37), we obtain the following expression for $b$-dependence of the anomalous magnetic moment

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(using the rescaling (3),(4), in order to give the physically correct order of magnitude):

$$\mu_B(b) = \frac{1}{c}\frac{C}{2\pi^2}\frac{\ln b}{b^2}\int_0^\infty dx \frac{\ln(x + \kappa \ln b)}{1 + x^2},\tag{42}$$

where the constant is $\kappa = 12\pi C \simeq 2.26$. We plot the $c$-unscaled anomalous magnetic moment $c\mu_B(b)$ versus $b$ in fig. 3. We observe that the anomalous magnetic moment first increases very quickly for $1 < b < 2$ to reach a maximum when $b \simeq 2$, and then decreases as $1/b^2$ for $b > 2$.

This behaviour can be tested experimentally, in case we apply such theoretical models to the physics of high temperature planar superconductors or, more generally, planar doped antiferromagnets. We reserve a detailed analysis of such physical applications for a future publication. However, for completeness we would like to give at this stage an estimate of the magnetic field for which the maximum of the anomalous magnetic moment occurs in the QED-like model for high temperature superconductivity of [7]. In those works we have a relativistic gauge system of nodal holon fermions coupled to a statistical gauge field, with a nodal velocity (playing the role of the speed of light) $\hbar v_F = 5 \cdot 10^{-4}c$, where $c$ is the real speed of light. The statistical gauge coupling $g^2$ is estimated to be [7] $g^2 \sim 4\pi\eta J\hbar v_F$, where $\eta$ is the doping concentration in the sample, and $J$ is the Heisenberg magnetic interaction. For maximum doping concentrations of order 10%, we have [7]: $\eta J \sim \mathcal{O}(10)$ meV, from which we obtain the estimate:

$$g^2 \sim 6.3 \cdot 10^{-15} \text{ GeV}\tag{43}$$

Thus, the maximum of $\mu_B$, which, as we infer from fig. 3, occurs for $b = \sqrt{eB}/g^2 \simeq 2$, is attained for magnetic fields of order $B \sim 6 \cdot 10^{-9}$ Tesla $= 0.06$ mGauss. These are still large compared to the order of the magnetically induced mass gaps (40) of fermion excitations at the nodes of this specific model [7]. Note that in condensed matter situations one uses external fields usually from the mGauss range up to 10 Tesla [7].

Experimentally, therefore, in order to test the predictions of this analysis on $\mu_B$, should the model be realised in nature, one should look at excitations near the nodes of the superconducting gap, within the superconducting (holon gapped) phase of a high-temperature superconducting planar material, and vary the externally applied magnetic field around the very low range of 0.06 mGauss. According to the model presented here, then, one should observe (c.f. fig. 3) a sharp increase (by an order of magnitude) and eventual decrease of the induced anomalous magnetic moment of the nodal holon

degrees of freedom. For larger magnetic fields, of the order often used in condensed matter applications, that is up to a few Tesla, the induced $\mu_B$ is considerably smaller, and there is a smooth dependence on the field intensity, as shown in fig. 3. We intend to come back to a detailed analysis of such issues, in the context of realistic condensed matter models, in a future publication.

# 4    Conclusions and Outlook

In this work we have discussed the anomalous magnetic moment of fermions in parity conserving $QED_3$, induced by the application of external fields. The induced term scales non trivially with the magnetic field, and when the latter is turned off to zero, one is left with a non trivial residual magnetic moment (19). On the other hand, in the presence of a strong field the induced magnetic moment scales non trivially with the field's intensity (38), (39).

These results may be of relevance to the physics of high-temperature superconducting materials, where, upon the application of a strong external magnetic field one observes indication for parity violation. In our model above we did not specify the physical nature of the dynamical "photon", which thus could be the statistical gauge field of [3, 7, 4] representing magnetic interactions (pairing) causing superconductivity. In fact we have been careful to keep its (three-dimensional) coupling $g^2$ different from the electromagnetic charge $e$ appearing in front of terms pertaining to the external field.

The induced parity violation is due to the magnetic moment term and not due to the mass gap of the fermions, which remains parity invariant. The derived scaling with the magnetic field can be tested experimentally. Moreover, the way we derived such a scaling, by means of inserting a mass for the dynamical "photon" in order to regularize IR divergences acquires itself a physical significance, once we apply the model to the physics of planar superconductors. Indeed, in models such as [3, 7], the statistical "photon", which in our discussion above appears with a coupling $g^2$, may become massive (with a non-perturbatively small mass) in the pseudogap phase, due to non perturbative effects [8], while it is exactly massless in the superconducting phase. The different scaling of the induced magnetic moment of the holons then between the two cases (c.f. [(8), (36)] and [(23),(38),(39)]), discussed in this work, may provide a way of testing such models experimentally.

There are many issues that we should look at in the near future. First of all, one should use realistic effective theories of high temperature superconductors, including the effects of holons. In a recent work [8] we have conjectured that there are regions in the parameter space of the condensed-matter models where spinons and holons exhibit (extended) supersymmetric dynamics. The physical degrees of freedom in such models are composites of spinons and holons, and they themselves exhibit supersymmetric spectra. In such cases one has both bosonic and fermionic charged excitations, which can both couple to the magnetic field. However the presence of the field breaks supersymmetry explicitly, due to the different ways by which bosons and fermions couple to a magnetic field. It would be interesting to study the induced magnetic moment in such broken supersymmetric cases, especially from the point of view of scaling with the external field. In fact, in four-dimensional particle physics, such computations reveal important signatures of supersymmetry, which can distinguish the Standard Model from, say, the Minimal Supersymmetric one. In a similar spirit, we expect in three dimensions important differences between the supersymmetric and non supersymmetric composite field theories describing the effective dynamics of (some regions of the parameter space of) antiferromagnets, which could be accessible to experiment. We plan to discuss such issues in the near future.

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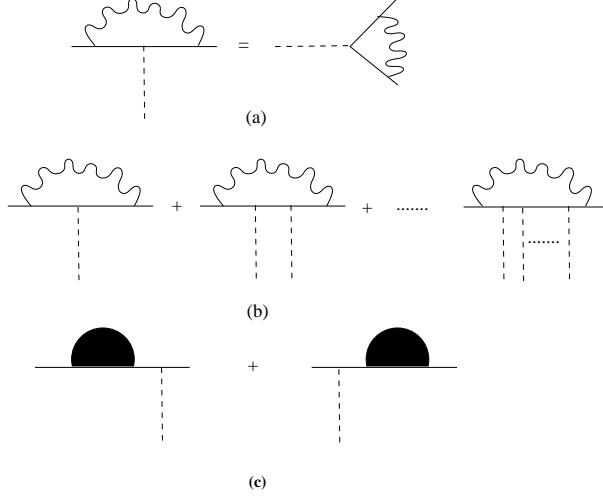
Figure 1: *One-loop graphs used for the computation of the anomalous magnetic moment. The fermions are represented by a continuous line, the dynamical gauge boson by a wavy line and the external field by a dashed line. (a) In the case of a weak external magnetic field, the fermion self energy corresponds to the vertex correction since there is only one external photon insertion. (b) In the case of a strong magnetic field, the fermion self energy corresponds to an infinite series of graphs, where one sums over the external field propagators. This summation is taken into account by the Schwinger proper-time representation of the fermion propagator in a classical electromagnetic field. (c) Self-energy corrections to the external fermion polarization spinors. Such reducible graphs should be added to (a) or (b), but are proportional to $\gamma^\mu$, and hence do not contribute to the anomalous magnetic moment. They provide finite renormalization of the vertex, canceling the terms proportional to $\gamma^\mu$ in (1). The dark-filled semicircles indicate dynamical gauge boson corrections with ((b)) or without ((a)) external field lines.*
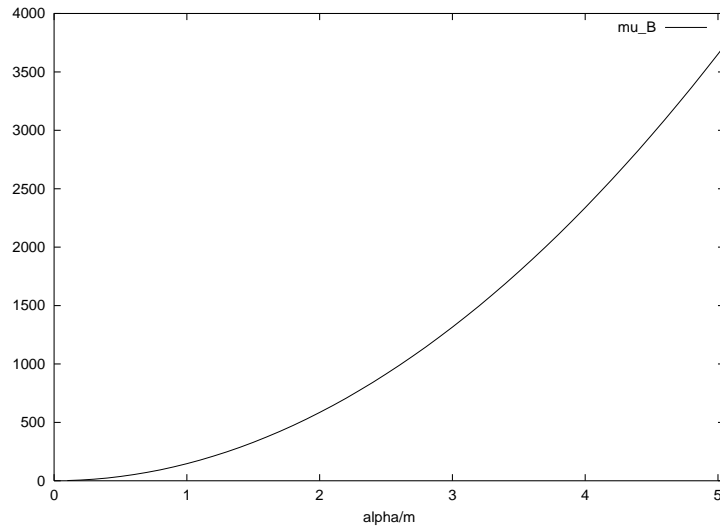
Figure 2: *The (c unscaled, for clarity in the plot) anomalous magnetic moment versus $\alpha/m$, for vanishing external field. The plot shows a quadratic dependence on $\alpha/m$. The physical values are obtained by the rescaling (4), with $c \sim 10^4$, and yield reasonable, smaller than one, values for $\mu_B^{\mathrm{phys}}$ in the physical regime of fermion masses $m/\alpha \sim 0.48$.*
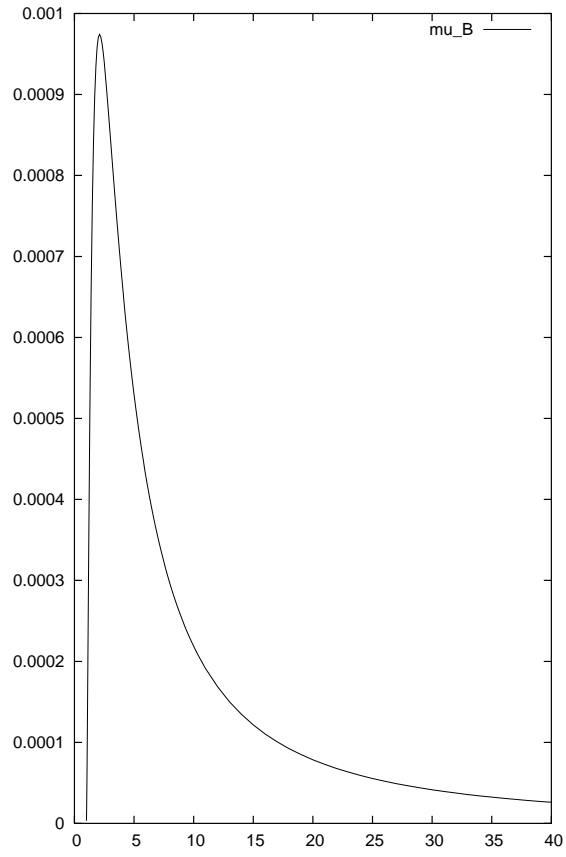
Figure 3: *The anomalous magnetic moment versus the dimensionless field strength b shows a maximum for $b \simeq 2$.*